1. **Communication Outcome: Students will read, write, listen to, and speak mathematics with understanding.**

   a. Students will read and listen to mathematical presentations and arguments with understanding. (AMATYC)

   b. Students will communicate both in speaking and in writing their understanding of mathematical ideas and procedures using appropriate mathematical vocabulary and notation. (AMATYC)

   c. Students will coherently communicate their own mathematical thinking to others. (NCTM)

**Level One Communication:**
The student is able to identify procedural steps presented in lecture or in the text.

When asked to explain a concept, the student will restate appropriate definitions, without applying them to a context, and use standard wording provided by the instructor or the text, without elaboration. The student has difficulty paraphrasing mathematical ideas and is unable to communicate a clear understanding of course concepts, though (s)he may be able to use these concepts to solve problems.

The student describes her/his mathematical thinking informally, without the use of conventional mathematical terminology or notation. (S)he shows her/his work in a haphazard manner with little organization or precision. Graphs, tables, and diagrams are poorly labeled.

**Level Two Communication:**
The student is able to identify and extract critical information from the lecture and the text.

The student can accurately paraphrase mathematical concepts in her/his own words. (S)he correctly uses common mathematical terminology.

The student’s solutions are clear, organized, and logical with correct use of mathematical notation. Her/his explanations are primarily procedural descriptions (“what”) without a rationale (“why”). Graphs, tables, and diagrams are properly labeled, though their use and relevance is not clearly explained.
Level Three Communication:

The student is able to identify, extract, and meaningfully organize the most critical information from the lecture and the text. (S)he reads the text with good comprehension. (S)he asks relevant questions to clarify ideas.

The student can accurately paraphrase mathematical concepts and give relevant examples that demonstrate a clear understanding of course concepts. (S)he correctly uses appropriate mathematical terminology and notation.

The student can clearly communicate her/his mathematical thinking using appropriate vocabulary and notation. (S)he supports her/his solutions by using short deductive chains of reasoning based on mathematical properties. Her/his explanations include a clear rationale. Her/his presentation is clear, organized, and concise. Graphs, tables, and diagrams are properly labeled and integrated into the explanation.

Level Four Communication:

In addition to Level Three abilities, the student self-monitors her/his understanding.

The student can explain mathematical concepts with clarity, accuracy, precision, and depth. (S)he uses illustrations from contexts outside of the classroom that clarify her/his explanations.

In addition to the Level Three abilities, (s)he is aware of and responsive to an audience. Her/his solutions are persuasive and elegant.

2. Problem-solving Outcome: Students will use mathematical reasoning to solve problems and a generalized problem solving process to work word problems.

a. The student can apply standard problem-solving methods and use relevant concepts to solve problems.

b. The student uses a generalized problem-solving rubric if such a rubric is used in the class.

c. The student’s written work demonstrates a conceptual understanding of course concepts.

d. The student’s written work supports his/her solution.

e. The student evaluates the reasonableness of his/her answer.
Level One Problem-solving:

a. The student can apply standard problem solving methods to solve problems that closely parallel problem types discussed in class and demonstrated in the text. The student has difficulty identifying an appropriate method if the context is changed in any but the most superficial way.

b. If a problem-solving rubric is used in class, the student addresses only some of the components of the rubric and his work is perfunctory. (For example, when asked to state the givens and the unknown, (s)he quotes the wording of the problem.)

c. The student’s work does not demonstrate a conceptual understanding of the ideas underlying the strategy. The student’s approach to problem-solving is algorithmic; her/his work essentially mimics a step-by-step process demonstrated in class or in the text.

d. The student is not always able to support her/his solution with clear work or descriptions.

e. The student is not able to demonstrate the reasonableness of her/his answer or to check her/his answer using standard methods. (S)he may check her/his work by reworking the problem a second time using the same strategy.

Level Two Problem-solving:

a. The student can apply standard problem solving methods to solve problems that differ in context from those discussed in class and demonstrated in the text if the given information is presented in a similar format.

b. If a problem-solving rubric is used in the class, the student addresses all of the components of the rubric, but in a cursory manner. (For example, (s)he identifies the givens and the unknown by quoting the wording of the problem. When asked to give a reasonable idea of the answer before solving, (s)he describes general characteristics of the answer like “it’s a number” or “its an expression with no negative exponents”.)

c. The student’s work does not demonstrate a clear understanding of the mathematical concepts that motivate her/his strategy.

d. The student’s work is organized but may lack explanations that show a logical connection between the given information and the answer.

e. The student can explain the reasonableness of her/his answer by using common sense if the context of the problem is familiar. (S)he uses standard methods to check her/his answers (e.g. plugging answer back into the original problem.)
Level Three Problem-solving:

a. The student modifies standard methods to solve problems that differ in context from those discussed in class and demonstrated in the text even if the given information is presented in a different format and includes extraneous information.

b. If a problem-solving rubric is used in the class, the student precisely addresses the components of the rubric. (For example, when asked to state the givens and the unknown, (s)he can precisely paraphrase the given information and define relevant variables. When asked to give a reasonable idea of the answer, (s)he can describe general characteristics of the answer and give a reasonable estimate.)

c. Student’s explanations demonstrate a good understanding of relevant mathematical concepts that motivate her/his strategies.

d. The student’s work is organized and shows a logical connection between the given information and the answer.

e. The student can explain the reasonableness of her/his answer by referring to the context of the problem and making logical inferences from the information given. (S)he checks her/his answers using standard methods. In situations where standard methods are not applicable, (s)he compares his answer to an estimate and reconciles differences between the two.

Level Four Problem-solving:

a. The student synthesizes and generalizes methods, without the aid of similar examples, to solve problems that require integration of two or more concepts.

b. If a problem-solving rubric is used in the class, the student precisely addresses the components of the rubric in-depth. (For example, (s)he can identify givens and the unknown as well as stating assumptions and conditions. (S)he analyzes her/his answer and her/his estimate and resolves differences.)

c. Student’s explanations demonstrate a thorough understanding of relevant mathematical concepts and clearly address the relationships between concepts. Her/his problem-solving strategies are efficient.

d. Her/his work is organized and logical; her/his explanations have both depth and breadth.

e. The student insightfully interprets solutions in the context of the problem and generalizes if appropriate. (S)he checks her work by using another method.
3. Use of Multiple Representations Outcome: Students will demonstrate the ability to use verbal, graphical, numerical, and symbolic representations of mathematical ideas.

   a. Students will use a variety of representations to demonstrate their understanding of mathematical concepts.
   
   b. Students will use a multi-prong approach to problem solving.
   
   c. Students will use appropriate technology to solve mathematical problems and judge the reasonableness of their results.

Level 1 Use of Multiple Representations:

*Multiple representations to show understanding:* When asked to explain a mathematical concept, the student tends to restate appropriate definitions without elaboration or reproduces an algorithmic and primarily symbolic process without a verbal discussion of the concept. (S)he does not tend to use graphs or other pictures to communicate her/his understanding of mathematical ideas. (S)he does not see the need for and does not use multiple representations to explain her/his thinking. The student has difficulty recognizing different representations of the same concept.

*Multiple representations to problem solve:* Student can read and use information that is given in a symbolic form or verbal form to solve standard problems that closely parallel presented examples. (S)he can organize information and analyze situations using a numeric table for situations that closely parallel presented examples, but (s)he has difficulty processing information given in graphical form. The student can represent simple mathematical situations symbolically and relies primarily on standard symbolic methods or numerical “guess and check” to solve problems.

*Technology:* The student correctly uses technology to perform numerical calculations to solve problems, but may have difficulty reconciling answers derived with a calculator to exact answers derived using a symbolic method. (S)he has difficulty using tables or graphs to estimate or check answers.

Level 2 Use of Multiple Representations:

*Multiple representations to show understanding:* The student can accurately paraphrase mathematical concepts in her/his own words. (S)he recognizes different representations of the same concept. If graphs, other diagrams, or tables are provided, (s)he can identify and use the relevant parts of these representations in her/his explanations of course concepts.
Multiple representations to problem solve: Student can use information that is given in a tabular, graphical, symbolic or verbal form to solve standard problems if the problems closely parallel previously seen examples and the information is presented in the same format. With explicit instructions, the student can create and use appropriate tables, graphs, or symbolic representations to solve problems. The student does not naturally apply a second method to check answers. If prompted, the student is able to use tables and graphs to estimate or check an answer derived by symbolic means.

Technology: The student uses technology to perform numerical calculations to solve problems. (S)he is able to reconcile answers derived with a calculator to exact answers derived using a symbolic method. If prompted, (s)he uses technology to generate tables and graphs in order to solve problems.

Level 3 Use of Multiple Representations:

Multiple representations to show understanding: The student produces and uses multiple representations to explain her/his understanding of mathematical ideas.

Multiple representations to problem solve: The student can use information that is given in a tabular, graphical, symbolic or verbal form to solve problems where the information is presented in a format different from previously seen examples. The student is able to transform information from one representation to another. The student identifies and applies an appropriate numeric or graphic or symbolic approach to solve problems without needing prompts in the instructions. The student utilizes an internalized “working” repertoire of multi-pronged strategies to solve problems. The student can produce conventional mathematical models by identifying essential elements of the context and devising representations that capture the mathematical relationships among these elements.

Technology: Without prompting, the student uses technology to generate numerical and graphical representations. (S)he uses technology to make estimates, solve problems, and check the reasonableness of answers derived using other methods.

Level 4 Use of Multiple Representations:

Multiple representations to show understanding: Student naturally and routinely uses multiple representations to explain her/his understanding of mathematical ideas. The student uses different representations in her/his explanations to illuminate different aspects of a complex mathematical concept or relationship.
Multiple representations to problem solve: The student routinely uses a multi-prong approach to problem solving by synthesizing information from two or more representations. Student chooses from among the various approaches and applies the most efficient method. (S)he demonstrates an understanding of the relative strengths and weaknesses of various representations for various purposes. (S)he routinely checks her/his work by using another representation.

Technology: (S)he uses technology to investigate mathematical ideas and solve problems in a manner that demonstrates an understanding of the limitations and capabilities of technology.

4. Effective Learner Outcome: Students will demonstrate the characteristics of an effective learner.

   a. Student has the will to succeed and demonstrates the characteristics of a successful student: motivation, responsibility, focus, perseverance, the ability to cope with anxiety, a good attitude toward learning, and time management skills.

   b. Student has the skills to succeed. (S)he uses appropriate resources to improve learning and reach goals.

   c. Student self-monitors and self-regulates. (S)he assesses personal strengths and weaknesses in his/her learning process and then seeks and implements a strategy for improving learning.

   Level One: the Dependent Learner

   Characteristics: The student depends on instructor’s reassurance and attention for motivation. (S)he has a negative attitude about learning math but recognizes that these attitudes affect her/his performance. (S)he acknowledges the need to try harder. (S)he attends class regularly, but his/her work is often incomplete. Procrastination and unrealistic scheduling interfere with academic success.

   Skills to succeed / use of resources: The student can state the location of Math Lab, instructor’s office, counseling services, and tutoring services but has only occasionally utilized these support services. The student is aware that the textbook is a resource but needs explicit instructions on how to use the text as a study aid. When study strategies are presented in class, the student does not see these strategies as relevant to his/her individual needs. The student has difficulty choosing and utilizing appropriate test preparation strategies.

   Self-regulation and self-monitoring: The student has little knowledge of his/her own learning process. (S)he expects the instructor to take the initiative in assessing his/her learning. Without personal attention from the instructor, (s)he
has difficulty focusing on the task at hand and gives up easily. (S)he is afraid to make mistakes. (S)he experiences feedback as a general affirmation or rejection of herself/himself. Emotional response to evaluation interferes with insight into her/his performance and impedes the development of a plan for improvement. (S)he judges her/his performance by how much effort it took.

**Level Two: the Trained Learner**

*Characteristics:* The student is motivated by her/his own academic goals, though (s)he does not see the value in specific course work outside of completing course requirements. (S)he sees math as a necessary hoop on the path to attaining her/his goals. (S)he shows initiative. (S)he follows instructions, remains focused, completes required tasks, and makes up missed work. (S)he asks for help when (s)he needs it. (S)he attends class regularly and completes assignments in a timely manner.

*Skills to succeed/use of resources:* The student can state the location of Math Lab, instructor’s office, counseling services, and tutoring services and utilizes these support services if they are advertised, discussed, or otherwise integrated into the classroom. The student uses the textbook as a resource if the text is consistently used as a reference in class. (S)he has difficulty identifying and choosing strategies that address her/his needs, but (s)he utilizes the study strategies and test-preparation strategies that are consistently reinforced in the classroom.

*Self-monitoring and self-regulation:* The student has knowledge of her/his own learning process and can articulate what factors help her/him learn. (S)he is able to use specific criteria to judge her/his own performance if this process is practiced in the classroom.

**Level Three: the Independent Learner**

*Characteristics:* The student is intrinsically motivated and persistent. (S)he has a good attitude toward math. (S)he seeks to understand the value of specific course work as it applies to situations beyond the assignment and outside of the math classroom. (S)he demonstrates the affective characteristics of a Level Two learner but also performs well in a less-structured classroom environment.

*Skills to succeed / use of Resources:* The student can state the location of Math Lab, instructor’s office, counseling services, and tutoring services and utilizes these support services on a regular basis. The student uses the text and other written materials as resources to deepen her/his understanding of concepts developed in the classroom. The student has internalized and effectively refined a variety of study strategies to fit her/his personal needs. Her/his approach to
learning is adaptive and efficient. (S)he participates in a study group outside of class.

*Self-monitoring and self-regulation:* The student has knowledge of his/her own learning process and can adapt it to new situations. (S)he monitors her/his own thinking, self-corrects, and learns from her/his errors. The student makes accurate self-assessments about his/her learning based on inferences drawn from general feedback. (S)he devises and implements appropriate strategies for improving her/his performance.

5. **Procedural Skills Outcome:** Students will recognize and apply math concepts in a variety of relevant settings and demonstrate the math skills and knowledge necessary to succeed in subsequent courses.