Unit 1

Linear Functions
Intermediate Algebra

Unit 1 Activity 0: Introduction to Numerical, Graphical, and Algebraic Modeling

update 2/6/06

Why: Mathematics can be used to create models of situations we want to study in “real life”. These models are useful for studying relationships, observing patterns, and making predictions. Mathematical models can be numerical, graphical, or symbolic. Each type of model has advantages and disadvantages.

Learning Objectives:
1. Communication:
   • Explain the mathematical reasoning supporting an answer.
   • Show work in a neat and organized manner.
2. Multiple Representations:
   • Use tabular, graphical, and symbolic models to answer questions.
   • Identify the advantages and disadvantages of using each type of model.

Warm-up:
1. Suppose that you are planning to borrow money to buy a car. What factors can influence the monthly payment?

2. Here is the graph of the monthly payments for a loan in terms of the amount of the loan, assuming the loan is paid off in 5 years at an 8.9% annual interest rate. Use the graph to answer the following questions.
   a. In problem #1 you listed factors that could affect the monthly payment on a loan. A graph is one way of modeling the relationship between two variables. Which two factors are used as variables in this graph?
   b. In a mathematical model, one variable is treated as the independent variable (the input) and the other as the dependent variable (the output). Which is the dependent variable in this scenario?
   c. Locate the point (10,000, 207.10) on the graph. Interpret the meaning of this point in the context of this scenario.
   d. Locate the point (10,000, 400). This point does not lie on the graph. How does this fact affect the way you would interpret the meaning of this point?
Group Exercise:
1. Use the graph to answer the following questions. Show on the graph how you determined each answer.
   a. Under these conditions, how much would your monthly payments be if you want to borrow $2,000? $7,000? $24,000?
   b. If you know that you can afford a $300 monthly payment, approximately how much money can you borrow?
   c. Suppose that you have already negotiated your loan, then you decide to borrow an additional $1,000, how much will the monthly payment increase? Briefly explain your reasoning.
   d. An algebra textbook says “Linear graphs have a constant rate of change”. What does this mean in this scenario?
   e. Make up a question dealing with car loans that could not be answered using this graph.
   f. What are the advantages and disadvantages of using a graph as a model.
2. The table relates the monthly payments to the original amount of a loan under the same conditions (5 year term with 8.9% annual interest). Use the table to answer the following questions. Briefly document your reasoning in an organized manner.

<table>
<thead>
<tr>
<th>Amount of loan</th>
<th>Monthly payments</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>41.42</td>
</tr>
<tr>
<td>4000</td>
<td>82.84</td>
</tr>
<tr>
<td>6000</td>
<td>124.26</td>
</tr>
<tr>
<td>8000</td>
<td>165.68</td>
</tr>
<tr>
<td>10000</td>
<td>207.10</td>
</tr>
<tr>
<td>12,000</td>
<td>248.52</td>
</tr>
<tr>
<td>14,000</td>
<td>289.94</td>
</tr>
<tr>
<td>16,000</td>
<td>331.36</td>
</tr>
<tr>
<td>18,000</td>
<td>372.78</td>
</tr>
<tr>
<td>20,000</td>
<td>414.20</td>
</tr>
</tbody>
</table>

a. Under these conditions, how much would your monthly payments be if you want to borrow $1,000?

$7,000?

$9,500?

$22,500?

b. If you know that you can afford a $300 monthly payment, approximately how much money can you borrow? Document your reasoning.

c. Describe any patterns you see in the table.

d. If you borrow an additional dollar, how much does your monthly payment increase?

e. Make up a question dealing with car loans that could not be answered using this table.

f. What are the advantages and disadvantages of using a table as a model?
3. The equation $R = 0.02071A$ gives the monthly payments, $R$, for a 5-year loan at 8.9% annual interest in terms of the amount of the loan, $A$. Use the equation to answer the following questions. Show your work.

a. What does the variable $R$ represent in this equation? What does the variable $A$ represent?

b. Under these conditions, how much would your monthly payments be if you want to borrow

   $10,000$?
   $9,500$?
   $25,249$?

c. If you know that you can afford a $250$ monthly payment, approximately how much money can you borrow? Show your work.

d. What do you think the number $0.02071$ in the equation tells us?

e. Make up a question dealing with car loans that could not be answered using this equation.

f. What are the advantages and disadvantages of using an equation as a model?
Intermediate Algebra

Unit 1 Activity 1: Modeling Linear Functions with Tables and Graphs

Updated 4/07/06

Why: In today’s world we have access to a lot of information. For example population estimates, demographic percentages, and statistics on housing costs can easily be found for your county. Numerical data about the stock market, inflation trends, and military spending is available to everyone on the Internet. It is easy to get overwhelmed with information. To make informed decisions based on information, we need to organize the information into a model that can be more easily understood and used. Algebra is a powerful tool for making models that are useful in making predictions. In this activity you will use tables and graphs to generate linear models.

Learning Objectives:
1) Communication: Show your work in a neat and organized manner. Explain your reasoning clearly. Clearly and precisely define variables. Interpret the meaning of slope and intercepts in the context of a problem situation.
2) Problem-solving: Follow directions. Apply modeling strategies to next scenarios. Check the reasonableness of answers.
3) Multiple Representations: Use information from tables and graphs to make an algebraic model (equation) and use the model to make predictions and decisions.
4) Skills: Find equations of lines.
5) Effective Learner: Work productively in a group. Connect mathematics to your own academic and career goals.

Warm-Up:
1) What is the general equation of a line? (We also call this the general form of a linear model.)

2) What does each of the letters in the equation represent?

3) Which of the letters are constants? What does it mean to say a letter represents “a constant”?

4) Which of the letters are variables? When you are asked to “define your variables” in a word problem, what does this mean?

5) Find the equation of the line that fits each description:
   a. has a slope of -2 and a y-intercept of 3
   b. has a slope of 1.5 and goes through (-2, -5)
Group Exercise:

1) AMATYC, the American Mathematical Association of Two Year Colleges, generated this table to model the percentages of community colleges offering distance learning in recent years.

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent</td>
<td>33</td>
<td>44</td>
<td>49.5</td>
<td>55</td>
<td>71.5</td>
</tr>
</tbody>
</table>

a. Can a linear equation be used to model this data? Explain.

b. Clearly define the variables you will use to find your model. (Hint: You will save time and have a more easily interpreted model if you think carefully about the “frame of reference” for the input variable.)

c. Find an algebraic model for this table.

d. Interpret slope in the context of this scenario.

e. Interpret the y-intercept in the context of this scenario.
2) The graph given is a model used by health care providers for finding ideal weight from a given height.

a) Why would health care professionals develop a model that correlates height and weight?

b) Which variable does this model treat as the dependent variable? Explain why this makes sense.

c) Do you think the model represents males and females? Do you think it represents adults of all nationalities? Explain.

d) Find an algebraic model for the relationship between weight and height. Define your variables precisely. (Hint: What “frame of reference” for the input variable might save you some time?)

e) Interpret the meaning of the y-intercept and the slope in your equation.

f) Use your equation to predict the ideal weight of a person who is 6’5” and of a person who is 3 feet. Which of these predictions seem reasonable? What does this tell health care providers about using this graph to find ideal weight?
Practice:
1) Corporations in the United States are doing quite well, thank you. The graph shows corporate profits, in billions of dollars, from 1988 through 2002.
   a) Interpret the point (3,520) in the context of this scenario.

   b) Find a linear model for the information shown in the graph. Clearly define the variables. Explain your reasoning or document your process so that another student can reproduce your strategy.

   c) Clearly explain the meaning of the slope in the context of the problem situation.

   d) Clearly explain the meaning of the vertical intercept in the context of the problem situation.

   e) Use your algebraic model to find the corporate profit in 1998.

   f) Use the graph to show that your answer in (e) is reasonable. Write a sentence and indicate directly on the graph how the graph shows that your answer is reasonable.

   g) Use your algebraic model to find when the profit was $700 billion.

   h) Use the graph to show that your answer in (g) is reasonable. Write a sentence and indicate directly on the graph how the graph shows that your answer is reasonable.
2) The table below gives the population of Suburbia over a number of years.

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>50,000</td>
<td>55,000</td>
<td>62,500</td>
<td>70,000</td>
<td>75,000</td>
<td>82,500</td>
</tr>
</tbody>
</table>

a) Can this table of data can be represented by a linear equation? Your classmate says “no” because linear equations have a constant rate of change, and in the table the population increases by 5,000 then by 7,500. Explain to your classmate what is wrong with his reasoning.

b) Find a linear model for the information shown in the table. Clearly define the variables. Explain your reasoning or document your process so that another student can reproduce your strategy.

c) Clearly explain the meaning of the slope in the context of the problem situation.

d) Clearly explain the meaning of the vertical intercept in the context of the problem situation.

e) Use your algebraic model to find the population in 1999.

f) Use the table to show that your answer in (e) is reasonable. Write a sentence and indicate directly on the table how the table shows that your answer is reasonable.

g) Use your algebraic model to find when the population is 70,600.

h) Use the table to show that your answer in (g) is reasonable. Write a sentence and indicate directly on the table how the table shows that your answer is reasonable.
i) Graph the data from the table. Be sure to clearly and completely label your graph.

<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>50,000</td>
<td>55,000</td>
<td>62,500</td>
<td>70,000</td>
<td>75,000</td>
<td>82,500</td>
</tr>
</tbody>
</table>

Tips for determining the window:
1) What is a reasonable XMIN? _________
   What is a reasonable XMAX? _________.
2) How many grid marks are on the x-axis in this grid? __________
3) To get a reasonable scale, calculate \( \frac{XMAX - XMIN}{\# \text{ grid marks}} \). Then round up to a number that is easy to work with.

Repeat this process for the y-axis.

j) Previously you found an equation to model the population of Suburbia. On the graph, show how the constants in the equation relate to the graph.

k) When did the population of Suburbia reach 80,000? Explain your reasoning or document your process so that another student can reproduce your strategy. Then show how to check your answer using another approach.

3) Explain how mathematical modeling could be used in a subject of interest to you or in your career/employment goal. Describe an example of what information is available (you don’t need to include any specific info – just a description) and what predictions or decisions you would want to make.
Why: One of the fundamental uses of Algebra is to model the world around us. Algebra provides the tools to translate a relationship we observe between two variables into an equation. We can then use that equation to make predictions or to analyze a situation. This activity is designed to help you learn to model linear relationships in real life contexts.

Learning Objectives:
2. Problem-solving: Construct models and use models to make predictions. Identify assumptions implied in a model.
3. Multiple Representations: Write an equation from a graph, a table, or a verbal description of a linear scenario.
4. Skills: Find the equation of a line from two points.
5. Effective Learner: Work effectively in a group.

Warm-up:
1. What is the general equation for a line?

2. What is the formula for the slope of a line?

3. Find the equation of a line fitting each description
   a. Has a slope of $\frac{1}{2}$ and contains the point (0,-3)
   b. Passes through the points (1,3) and (0,4)
   c. Passes through the points (1,3) and (-2,4).
Group Exercise:
1. A biologist takes several measurements of the concentration of contaminant in a lake after a chemical spill. After 4 hours, she measures 450 milligrams of contaminant per milliliter. After 12 hours, the concentration has decreased to 60 mg/ml.

   a. Clearly define the variables. How do you know which is the input (independent variable) and which is the output (dependent variable)?

   b. Write the information given as points (coordinate pairs) or make a table if you prefer.

   c. Find an algebraic model for this scenario.

   d. Interpret the meaning of the slope in the context of this scenario.

   e. Use your model to estimate the time at which there is no more contamination in the lake.

   f. What assumption is the biologist making if she uses your model to make predictions?

   g. Interpret the meaning of the y-intercept in the context of this scenario.
2. While Mutual Funds have been in the news recently for the “questionable behavior” of some fund managers, other Mutual Funds continue to be successful. One example is the Fidelity Magellan Fund, which has been in existence over 44 years. In 1997, the net assets of the Fidelity Magellan Fund were $51.243 billion, and in 2001 they were $80.190 billion. A minimum investment of $2,500 is required for first time investors, though the minimum investment for an IRA is $500. Assuming that the assets of Fidelity Magellan continue to grow at the same rate, what will be the net worth of this fund in 2010?

a. Paraphrase in a very clear statement the question you are to answer.

Write down two very clear statements of what you want your variables (input and output) to represent. How did you decide what the variables should represent?

b. Some of the information given will not be useful in finding a model or in answering the question asked. Cross out the extraneous information in the problem.

c. Circle the wording that indicates a linear model is appropriate.

d. Write the relevant information as points (or as a table if you prefer). Then find the linear model and clearly define your variables.

e. Use your model to answer the question originally asked in the problem.

f. When will the Magellan fund reach $100 billion? What assumption underlies your answer?

g. Sketch a graph of your model. Clearly scale and label your axes.

h. Interpret the slope of your equation in the context of this scenario.
Practice:

1. According to the World Almanac and Book of Facts, the life expectancy of US males increased from 47.3 years in 1900 to 73.9 years in 1998. Life expectancy is how long the average male is expected to live. The average height of males in the U.S. has also increased over the last century. Men are now approximately 10 inches taller on average than they were in 1900. Do you know anyone who was born around 1900? Assume the male life expectancy increased at the same yearly rate. How long will men live on average in the year 2050?

   a. Paraphrase in the question you are to answer.

   Write down two clear statements of what you want your variables to represent. Explain how you made your choice.

   b. Some of the information given will not be useful in finding a model or in answering the question asked. Put an X through extraneous information in the problem.

   c. Circle the wording that indicates a linear model is appropriate.

   d. Represent the relevant information as points or in a table. Then find the linear model.

   e. Use your model to answer the question originally asked in the problem.

   f. Sketch a graph of your model. Clearly scale and label your axes.

   g. Use your graph to estimate when the life expectancy for U.S. males was 65.

   h. Interpret the slope of your equation in the context of this scenario.
Intermediate Algebra

Unit 1 Activity 3: Problem-solving practice with linear modeling

Why: Learning to use a general problem-solving process is one of the major goals of Intermediate Algebra at LMC. This activity will prompt you through the steps of solving a real-life problem that contains extraneous information.

Learning Objectives: In this activity students will work on the following skills connected to the problem-solving process:
1. Identify given information and extract extraneous information
2. State assumptions necessary to solve the problem
3. Make reasonable estimates and document your thinking
4. Make and use a linear model to make a prediction
5. Interpret your answer in context.

In 1985 the United States consumed 12.28 quadrillion BTU in coal. Our consumption had risen to 19.40 quadrillion BTU by 1994. One BTU (British thermal unit) is the amount of energy required to raise the temperature of one pound of water one degree Fahrenheit at or near 39.2 degrees Fahrenheit. One quadrillion, 10E15, BTU is equivalent to 26 hours of world energy use in 1989. If these trends continue, what is the predicted coal consumption for the U.S. this year?

a. Paraphrase the task you are asked to perform.

b. Circle the information in this problem that will be used to answer the question. Cross out extraneous information.

c. State assumptions you need to make in order to model this data.

d. Make a reasonable estimate and document your estimation process.

e. Solve the problem using any method you choose. Clearly define variables if you set up an equation.

f. Write a sentence interpreting your answer in the context of the problem.
Intermediate Algebra

Unit 1 Activity 4: Introduction to the concept of a function

Why: This activity introduces you to the concept of a function. As you have seen in previous activities, mathematical models help us see relationships between two variables. A function is a special type of relationship. Most mathematical models are functions. For example, in Intermediate Algebra, you will encounter linear, exponential, and quadratic functions. In transfer-level math classes, you will be expected to use and interpret function notation.

Learning Objectives:
1. Communication:
   • Identify functions in a variety of everyday contexts and explain why the relationships are functions.
   • Interpret function notation in a context

2. Multiple Representations:
   • Identify, use, and evaluate functions in verbal descriptions, tables, and graphs.
   • Find algebraic models for linear functions given a table or a graph.

Definition: A function is a relationship in which each input is associated with one and only one output.

You can think of a function as an “input-output” machine. For the picture below y is a function of x. When y is a function of x, this means that y depends on x.

<table>
<thead>
<tr>
<th>Input (x)</th>
<th>f for function machine</th>
<th>Output y = f(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Independent variable</td>
<td>f</td>
<td>Dependent variable</td>
</tr>
</tbody>
</table>

Warm-up: An investigation of the concept of a function

1. Using a verbal description to identify a function
   a. Consider the relationship of LMC student ID numbers and student social security numbers. Are social security numbers a function of student ID numbers? Explain. (Hint: Remember that “y is a function of x”, so which variable is the input or independent variable?)

   b. Consider the relationship of LMC student ID numbers and student phone numbers. Are phone numbers a function of student ID numbers? Explain. (Hint: First determine which variable is the input or independent variable.)
2. **Introduction to function notation**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Population of Town A in thousands</td>
<td>125</td>
<td>145</td>
<td>175</td>
<td>205</td>
<td>225</td>
<td>255</td>
</tr>
</tbody>
</table>

a. Use the definition of a function and the table to explain why *population* is a function of the *year*.

b. Test your understanding of the definition of a function: change one value in the table so that *population* is NOT a function of the *year*.

c. Define $x$ to be years from 1990 and $y = f(x)$ to be the population of Town A in thousands.
   i.) What is the value of $f(2)$? What does $f(2)$ mean in the context of this scenario?

   ii.) Estimate the value of $f(9)$. Explain your reasoning. What does $f(9)$ mean in the context of this scenario?

d. Is *population* a linear function of the *year*? Support your answer.

e. Find a linear equation $y = f(x)$ for population data in the table above. Define your variables.

f. Use your equation to find $f(-5)$. Interpret this value in the context of the problem.

f. Use your equation to find the value of $x$ for which $f(x) = 205$. Interpret this value in the context of the problem.
**Group Exercise:**

1. Define $t$ to be years after 1990 and $P = g(t)$ to be corporate profit in billions.

   a. In the graph shown, corporate profit is a function of the year. How could you change the shape of the graph so that profit is NOT a function of the year? Draw it on the grid.

   b. What is the value of $g(5)$? What does $g(5)$ mean in the context of this scenario?

   c. Estimate the value of $g(-2)$. What does $g(-2)$ mean in the context of this scenario?

   d. Find a linear equation for profit data in the graph above.

   e. Use your equation to find $g(14)$. Interpret this value in the context of the problem. Show how to use the graph to check your answer.

2. A look at postal rates . . .

   a. When you mail a package, what are some of the factors that influence the cost?

   b. If material is mailed in a Priority Mail Flat Rate Envelope, the cost is $3.85, regardless of weight or destination. So when using a Priority Mail Flat Rate Envelope, obviously cost is not determined by the weight. Now think like a mathematician, is cost a function of weight? Explain.
c. The following information was taken from http://www.usps.com. Priority Mail is used for documents, gifts, and merchandise. Any mailable item may be sent as Priority Mail. The maximum weight is 70 pounds. The maximum size is 108 inches or less in combined length and distance around the thickest part.

**Priority Mail Rates**

<table>
<thead>
<tr>
<th>Weight not over (pounds)</th>
<th>Local or Zones 1-3</th>
<th>Zone 4</th>
<th>Zone 5</th>
<th>Zone 6</th>
<th>Zone 7</th>
<th>Zone 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$3.85</td>
<td>$3.85</td>
<td>$3.85</td>
<td>$3.85</td>
<td>$3.85</td>
<td>$3.85</td>
</tr>
<tr>
<td>2</td>
<td>3.95</td>
<td>4.55</td>
<td>4.90</td>
<td>5.05</td>
<td>5.40</td>
<td>5.75</td>
</tr>
<tr>
<td>3</td>
<td>4.75</td>
<td>6.05</td>
<td>6.85</td>
<td>7.15</td>
<td>7.85</td>
<td>8.55</td>
</tr>
<tr>
<td>4</td>
<td>5.30</td>
<td>7.05</td>
<td>8.05</td>
<td>8.50</td>
<td>9.45</td>
<td>10.35</td>
</tr>
<tr>
<td>5</td>
<td>5.85</td>
<td>8.00</td>
<td>9.30</td>
<td>9.85</td>
<td>11.00</td>
<td>12.15</td>
</tr>
</tbody>
</table>

i.) When mailing a package using priority mail, is cost a function of weight? Explain.

ii.) Assume that a package fits the size restrictions for using Priority Mail. Let \( C = f(w) \) be the cost of mailing the package via priority mail to Zone 4. What is the value of \( f(1.5) \) and what does this value mean in the context of this scenario?

**Practice:**

1. Barry Bonds, who plays for the San Francisco Giants, now holds the major league single-season home run record. His home run counts for 1986 to 2001 are listed below.

<table>
<thead>
<tr>
<th>Year</th>
<th>Home Runs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1986</td>
<td>16</td>
</tr>
<tr>
<td>1987</td>
<td>25</td>
</tr>
<tr>
<td>1988</td>
<td>24</td>
</tr>
<tr>
<td>1989</td>
<td>19</td>
</tr>
<tr>
<td>1990</td>
<td>33</td>
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<tr>
<td>1991</td>
<td>25</td>
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<td>1992</td>
<td>34</td>
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<tr>
<td>1993</td>
<td>46</td>
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<td>1994</td>
<td>37</td>
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<td>1995</td>
<td>33</td>
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<td>1996</td>
<td>49</td>
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<td>1997</td>
<td>40</td>
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<tr>
<td>1998</td>
<td>37</td>
</tr>
<tr>
<td>1999</td>
<td>35</td>
</tr>
<tr>
<td>2000</td>
<td>49</td>
</tr>
<tr>
<td>2001</td>
<td>73</td>
</tr>
</tbody>
</table>

For Barry Bonds, is his home run count a function of the year? Explain.

2. Draw a graph where \( y \) is NOT a function of \( x \).

3. Make a table where \( y \) is NOT a function of \( x \).

4. If \( h(x) = \frac{2}{3}x - 4 \), find the value of \( h(0) \), \( h(6) \), and \( h(1) \).
5. An organization that advocates internationally on women’s issues is studying the relationship of education and a variety of health statistics.

The graph to the right shows the function \( M = h(p) \), where \( p \) is percentage of adult females who are literate in a country and \( h(p) \) represents the mortality rate, measured in deaths per 100 thousand, for children under five in the country.

a. What does the graph indicate about the relationship between women’s literacy and the mortality rates of young children?

b. What are some factors that might account for the relationship between these two variables?

c. Find the value of \( h(50) \) and interpret the meaning of the point \((50, h(50))\) in the context of this model.

d. Find the value of \( p \) for which \( h(p) = 50 \) and interpret the meaning of the point \((p, 50)\) in the context of this model.

e. Find the equation for this function. Clearly define your variables.

f. Find the horizontal intercept for this function. Does this point have meaning in the context of this scenario? Explain.
**Why:** This activity is designed to provide practice with using a general problem-solving process in the context of linear modeling and interpreting function notation. These problems will help students review for the next exam.

**Learning Objectives:**
1. Problem-Solving: Demonstrate the use of a general problem-solving process. State assumptions, make estimates, solve problems, and analyze the “reasonableness” of an answer.
2. Applications and Skills:
   a. Use a mathematical model to answer questions and make predictions.
   b. Use, evaluate, and interpret function notation in tables, graphs, and symbolic form.
3. Communication: Interpret the meaning of solutions in the context of the model.

**Group Exercise:**
1. Suppose that Ann is watching her weight. She carefully weighs her food and calculates the calories using the nutritional information given on the packaging. The calories in her breakfast cereal are a linear function of the weight of the cereal, given in tenths of a gram. The function is 
   \[ C = f(w) = 23w, \]
   where \( w \) is the weight in tenths of a gram and \( C \) is the calorie count.
   
   a. Find the value of \( f(8) \) and write a clear and complete English sentence explaining what the value means in the context of this problem situation.

   b. Find \( w \) such that \( f(w) = 287.5 \) and write a clear and complete English sentence explaining what the value means in the context of this problem situation.

   c. Find the slope of the function and write a clear and complete English sentence explaining what the value means in the context of this problem situation.
Practice:
1. The cost of a phone call to China is given by \( C = g(t) = 1.75 + .60(t - 1) \), where \( t \) is the length of the call in minutes.

   a. Find \( g(6) \) and write a clear and complete English sentence explaining what the value means in the context of this problem situation.

   b. Find \( t \) such that \( g(t) = 5 \) and write a clear and complete English sentence explaining what the value means in the context of this problem situation.

   c. Find the slope of the function and write a clear and complete English sentence explaining what the value means in the context of this problem situation.

2. In 1980 the U.S. had half a million people in prison. This same year, roughly 80\% of the inmates were male with approximately half imprisoned for drug-related offenses. Twenty years later the number of prisoners in the U.S. had grown to 1.9 million, but the proportion convicted of drug-related offenses remained approximately the same. In the year 2000 women still comprised 20\% of the adult convict population.

   Write an equation that models this situation. Use the equation to find the year when the U.S. had one million prisoners.

   a. Paraphrase the tasks you are asked to perform.

   b. Paraphrase the important information in this problem.
c. List extraneous information.

d. State assumptions you need to make in order to model this data.

e. Make a reasonable estimate for the year when the U.S. has one million prisoners and document your estimation process.

f. Solve the problem using any method you choose. Clearly define variables.

g. Write a sentence interpreting your answer in the context of the problem.

h. Explain why your answer is reasonable.
Unit 1 Activity 6: Introduction to Linear Regression on a Graphing Calculator

**Why:** Real data is often not perfectly linear. But if the data is roughly linear, a linear model may be a desirable choice. Graphing calculators can be used to find the linear model that best fits the data. Finding the line of best fit is called linear regression. All calculators will give the same regression line for a set of data. The regression line is the line that best fits the data. It passes as close as possible to all of the data points. No other line will have the data as closely grouped about it as the regression line. So the regression line gives the best predictions possible based on the data. This activity will introduce you to regression modeling with a TI graphing calculator.

**Learning Objectives:**
1) Communication: Interpret the meaning slope, intercepts, and solutions in the context of the model.
2) Problem-Solving: State assumptions, make estimates, and analyze “reasonableness” of an answer.
3) Multiple Representations: Enter data on a graphing calculator and use linear regression to find the equation of the best linear model.
4) Applications and Skills: Use the model to answer questions and make predictions.

**Warm-Up:**
1) The graph illustrates the relationship between years of education and mean personal yearly income for a sample 1000 adults in the U.S. The line drawn is the regression line; it is the line that best fits the data. It’s equation is $f(x) = 2567.07x - 4512.66$. This regression equation was derived using regression capabilities of a graphing calculator.

   a. Interpret the point (4, 10,000) in this context.

   b. What does this model predict is the yearly mean income for people with an AA degree? Show how you determined your answer. What *assumptions* are you making?

   c. Give an example of a number of years of education for which the model overestimates the average annual income seen in the actual data.
d. Give an example of a number of years of education for which the model underestimates the average annual income seen in the actual data.

e. Interpret the slope of the regression line in the context of this problem.

f. The regression line may or may not actually pass through the data points, so the calculator is clearly using a different process than the one you use when finding an equation of a line through two points. To emphasize this point, see if you can choose two data points that form a line
   i. with the largest possible slope
   ii. with the smallest possible slope
   iii. with a slope equal to the regression line

Group Exercise

1. The following set of data gives the IQ scores for five 2nd graders and the number of hours they watch TV each week.

<table>
<thead>
<tr>
<th>Hours of TV viewing per week</th>
<th>10</th>
<th>12</th>
<th>8</th>
<th>2</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>IQ</td>
<td>110</td>
<td>105</td>
<td>120</td>
<td>140</td>
<td>100</td>
</tr>
</tbody>
</table>

a. Suppose that a psychologist studying the effects of TV on intelligence decides to model the data using a linear equation.
   Which variable do you think she will want to use as the dependent variable? Why?

b. Do you think the linear regression model will have a positive slope or a negative slope? Explain.

c. Make a scatterplot of this data in the grid provided.
   Clearly label and scale your axes.
d. **Follow the instructions on the last page to find the linear regression equation for this data.** Write down the information the calculator is giving to you.

Use this information to write the equation of the regression line. Clearly define your variables.

e. **Graph your regression line on the scatterplot of the data you drew above.**

   Tip: Make a table using your regression equation with at least three points. Plot the points on the scatterplot and draw the regression line through them.

f. **Use the equation of the regression line to answer the following questions:**

i. What is the slope and what does it mean in this context?

ii. What is the y-intercept and what does it mean in this context?

iii. If a child’s IQ is 135, predict the number of hours she watches TV a week. Show this point on your graph.

iv. If a child watches 15 hours of TV each week, what IQ does this model predict he will have? Show this point on your graph.
Practice:

1. The table below is from the National Center for Health Statistics.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>% of adults who smoke</td>
<td>42.4</td>
<td>37.4</td>
<td>34.1</td>
<td>33.2</td>
<td>30.1</td>
<td>25.5</td>
<td>24.7</td>
</tr>
</tbody>
</table>

a. BEFORE you use your calculator to find a linear model, do you think the slope will be positive or negative? Explain.

b. Sketch the scatterplot of this data in the grid provided. Clearly label and scale your axes.

c. Find the regression line that models this data. Clearly define your variables.

d. Draw your regression line on the scatterplot.

e. Someone in your group suggests that you check your regression equation by plugging in values from the table. Is this a reasonable way to check the accuracy of a regression equation? Why or why not?

f. Interpret the meaning of the y-intercept in the context of scenario.

g. Interpret the meaning of the slope in the context of scenario.

h. Use your model to estimate the percentage of adults who smoke this year.

i. According to your model, when will adults in the U.S. no longer smoke?
Regression is a concept you will study further if you take Statistics. Finding a regression equation involves three steps in the statistical menu of the calculator:
1) entering the data as lists (EDIT),
2) choosing the type of regression equation (in this case a linear equation) (CALC),
3) telling the calculator which data list is the independent variable (input) and which data list is the dependent variable (output).

Follow the instructions below to find the linear regression equation.

<table>
<thead>
<tr>
<th>TI-83 Plus</th>
<th>TI-86</th>
<th>TI-89</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>STAT</strong></td>
<td><strong>Choose STAT by pressing $2^{nd}$ +</strong></td>
<td><strong>APPS</strong></td>
</tr>
<tr>
<td>Choose <strong>EDIT #1</strong> or press</td>
<td>Choose Edit <strong>F2</strong></td>
<td>#1 FlashApps</td>
</tr>
<tr>
<td><strong>ENTER</strong></td>
<td></td>
<td>Stats/List Editor</td>
</tr>
<tr>
<td>Enter all $x$-values in L1</td>
<td>Enter the $x$-values in $x$-stat and the $y$-values in $y$-stat. Enter a 1 for each data point in $f$-stat (this indicates to the calculator that each data point is used only once.)</td>
<td>Type a name into a blank space in the top row or clear a list by hitting CLEAR when the cursor is highlighting the list name. Move cursor down and list will clear. Enter all $x$-values in one list and $y$-values in another.</td>
</tr>
<tr>
<td>and all $y$-values in L2.</td>
<td></td>
<td>F4 Calc</td>
</tr>
<tr>
<td><strong>STAT</strong></td>
<td>**Choose CALC (Use Right arrow), press <strong>ENTER</strong></td>
<td>#3 Regressions</td>
</tr>
<tr>
<td>Choose <strong>CALC (a+b)</strong> #4</td>
<td>**Choose <strong>F1</strong></td>
<td>#1 LinReg (a+bx)</td>
</tr>
<tr>
<td>Type the name of the $x$-list, comma, name of the $y$-list. L1 is $2^{nd}$ L1, L2 is $2^{nd}$ L2.</td>
<td>**Choose <strong>LinR</strong> <strong>F3</strong> <strong>F3</strong></td>
<td>#2 LinReg (ax+b)</td>
</tr>
<tr>
<td><strong>ENTER</strong></td>
<td><strong>ENTER</strong></td>
<td>Type in the name of each list or choose VAR-LINK ($2^{nd}$) and scroll down to list name and hit ENTER.</td>
</tr>
</tbody>
</table>

(* To clear data in a list, put the cursor on the list name by using the up arrow. Hit CLEAR. Put cursor on the first entry in the list by using the down arrow.)

** You can name a list by moving the cursor to a blank space in the top row. Type the name. (Note the A in the cursor; the calculator is already in ALPHA mode.) Move cursor down below the name to enter the data.

** LinR automatically uses data in $x$-stat and $y$-stat. To use data in other lists, press **LinR F3**, then choose **LIST** by pressing $2^{nd}$ . Choose **NAMES F3**, then hit the $F$ key corresponding to the name of your $x$-list, then comma, then the $F$ key corresponding to the name of your $y$-list.
Why: Previously, we have seen many examples where two quantities have a linear relationship. For example, the amount spent on gasoline and the gallons purchased have a linear relationship. We have found linear equations to model these relationships and used the equations to make predictions. Now we will encounter situations where two linear equations will be used to make a comparison. For example, we might want to compare two gas stations and see how much gas we could buy for the same amount of money. In this situation we will have two unknown quantities that we want to find. If there are two unknown quantities, there are two variables and we need two equations in order to find a solution. In this activity, we will practice using tables, graphs, and equations to solve a system.

Learning Objectives:
1. Multiple Representations:
   a. Solve a system of equations using a tables, graphs, equations and technology.
   b. Model data, graphs, and verbal descriptions with linear systems and solve problems using the models.
2. Problem-solving:
   a. Make estimates in order to solve a problem.
   b. Check answers using another method.
3. Communication:
   a. Explain your mathematical reasoning.
   b. Interpret solutions in the context of the problem.

Warm-up:
1) The two lines in the graph represent the growing profits of companies A and B in thousands of dollars. Suppose that $x$ is the number of years from 2000.
   a) The profits of Company A are growing at a slower rate than Company B. Label the lines.
   b) When will both companies have the same profit?
   c) What will that profit be?

2) Now let's look at problem #1 from the point of view of a system of equations.
   Here are two equations describing the profit in terms of years: $y = 10x + 35$  $y = 15x + 20$
   a) Match each equation with a company. Explain your reasoning.
   b) According to the graph, what is the solution to this system of equations?
   c) Check that your solution makes both equations true.
   d) Write a sentence that explains what your solution means in the context of this problem.
**Group Exercise:**

1) Revenue for a student career conference in 1999 came from a $200 donation from a local sponsor and a $10 registration fee for each student. The group organizing the conference paid $450 for printing promotional flyers and provided lunch at a cost of $5.50 for each lunch.

a) Write an equation that represents the revenue made by the conference and an equation that represents the cost to put on the conference. Define your variables.

b) Graph each equation on your graphing calculator. Reproduce a neat sketch of your graphs on the grid. Clearly label and scale your axes.

c) Follow the instructions at the end of this activity to use your calculator to find the intersection point. Indicate the coordinates of the intersection point on your graph.

d) The break-even point is where cost equals revenue. How many people attend the conference if the group breaks even? How much profit do they make?

2) Ann calls her Swedish aunt 12 times a year. Her calls usually last between 20 minutes and 2 hours. Suppose AT&T charges $6.01 for a 7-minute call to Sweden and $6.79 for an 8-minute call to Sweden. Suppose that MCI charges $0.66 a minute plus a $1.99 overseas connection fee.

a) Write an equation that represents each company’s charge. Define your variables.

b) Use an algebraic method to determine the length of a phone call that results in the same charge for both companies. State your answer in a sentence that relates to the context of the problem.

c) Which phone company do you recommend Ann to use? Support your recommendation.
3) Consider the system \( x + y = 40 \) \( 0.32x + 0.19y = 12.15 \)

a) In the table Y1 is y-values for \( x + y = 40 \) and Y2 is y-values for \( 0.32x + 0.19y = 12.15 \)

<table>
<thead>
<tr>
<th>X</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y1</td>
<td>40</td>
<td>30</td>
<td>20</td>
<td>10</td>
<td>0</td>
<td>-10</td>
</tr>
<tr>
<td>Y2</td>
<td>63.95</td>
<td>47.11</td>
<td>30.26</td>
<td>13.42</td>
<td>-3.42</td>
<td>-20.26</td>
</tr>
</tbody>
</table>

Use the table to estimate the solution to the system. Explain how you determined your estimate.

b) Now solve the system using algebra.

c) Comment on the accuracy of your estimate.

**Practice:**

1) You are in the market for a cell phone. On a website comparing three cell phone companies, you find this graph. Use the graph to answer the following questions.

a) When will the monthly cost for Horizon and UTT be equal? What will the cost be?

b) If you talk an average of 10 hours a month, which company offers the cheapest deal?

c) What does Horizon charge per minute?

d) Describe the phone usage of a customer who will benefit from choosing Singular.

e) If you only plan to use your cell phone for emergencies, which company will you choose? Why?

f) Based on the graphs given, which company would you choose? Why?
2) The same website compares the monthly cost of three other companies using the following table.

<table>
<thead>
<tr>
<th>Minutes</th>
<th>0</th>
<th>30</th>
<th>60</th>
<th>90</th>
<th>120</th>
<th>150</th>
<th>180</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABC</td>
<td>0</td>
<td>9</td>
<td>18</td>
<td>27</td>
<td>36</td>
<td>45</td>
<td>54</td>
</tr>
<tr>
<td>SKY-1</td>
<td>15.00</td>
<td>17.70</td>
<td>20.40</td>
<td>23.10</td>
<td>25.80</td>
<td>28.50</td>
<td>31.20</td>
</tr>
</tbody>
</table>

a) Use the table to estimate when ABC charges the same amount as CELL-1. Estimate the monthly cost at this point. Explain your reasoning.

b) Use the table to estimate when ABC charges the same amount as SKY-1. Estimate the monthly cost at this point. Explain your reasoning.

c) Describe the phone usage of someone who benefits from using ABC. Be specific.

d) Use the table to find the monthly cost equation for each cell-phone company.

e) Use your equations to rework part (b). How reasonable was your estimate?

f) Describe the phone usage of someone who benefits from CELL-1. Be specific.

g) Which of these phone companies would you choose? Why?
Key- strokes for using the Intersection Button on a TI Graphing Calculator:

<table>
<thead>
<tr>
<th>TI – 83</th>
<th>TI – 86</th>
<th>TI – 89</th>
</tr>
</thead>
</table>
| After entering the equations in **Y=** and setting a window, choose **CALC** by pressing **2nd** [**Trace**] **#5 Intersection**
*First Curve?* [If cursor is on one of the lines you are working with, hit **ENTER**. Otherwise, use arrows to move cursor to line you want and hit **ENTER**.]
*Second curve?* [If cursor is on the other line in the system, hit **ENTER**. Otherwise, use arrows to move cursor to second line and hit **ENTER**.]
*Guess?* [Move cursor “close to” point of intersection] **ENTER**

| Enter the equations and set a window. **Graph** menu
| Hit **MORE** to get to the **MATH** submenu.
| Hit **MORE** and choose **Isect** **F3**
| *First Curve?* [If cursor is on one of the lines you are working with, hit **ENTER**. Otherwise, use arrows to move cursor to line you want and hit **ENTER**.]
| *Second curve?* [If cursor is on the other line in the system, hit **ENTER**. Otherwise, use arrows to move cursor to second line and hit **ENTER**.]
| *Guess?* [Move cursor “close to” point of intersection] **ENTER**

| Enter the equations and set a window. **Graph** menu.
| Choose the **Math** submenu by hitting **F5**.
| **#5 Intersection**
| *1st curve?* [If cursor is on one of the lines you are working with, hit **ENTER**. Otherwise, use arrows to move cursor to line you want and hit **ENTER**.]
| *2nd curve?* [If cursor is on the other line in the system, hit **ENTER**. Otherwise, use arrows to move cursor to second line and hit **ENTER**.]
| *Lower Bound?* [Move the cursor so the x-coordinate is to left of x-value of the intersection point, press **ENTER**];
| *Upper Bound?* [Move the cursor so the x-coordinate is to right of x-value of the intersection point, press **ENTER**];
Why: Throughout this unit you have been using the steps in a general problem-solving process to solve “real-life” problems. In this activity you will again practice the general “habits of mind” that are vital to solving messy real world problems. Specifically, you will need to weed out extraneous information, create useful representations of information in order to solve a problem, document your reasoning as you solve a problem, and interpret the results.

Learning Objectives:
Problem-solving: Use the steps of a general problem-solving process to
1. identify important information and paraphrase the task
2. identify, generate and use multiple representations to solve the problem,
3. interpret your solution in a manner appropriate for a specified audience.

The Collision Problem: You have been hired by an insurance investigator to look into an accident involving a sailboat and a speedboat that occurred on New Year’s day at 3:00 in the afternoon. Your job is to determine where (relative to the dock) the collision took place. After your investigation you ascertain the following facts about the case. The speedboat was carrying a group of 20 people who were returning to shore after enjoying a 2-hour picnic on an island. The island is located 5.2 miles east and 12.6 miles north of the dock. The speedboat was heading from the island toward a point on the shore located 3.0 miles due west of the dock. Around the same time, the sailboat left the dock heading toward a buoy that is 5.0 miles west and 7.5 miles north of the dock. Neither boat made it to its destination since they collided in the water. You interviewed 5 witnesses and 2 lifeguards for 4 hours, in addition to studying nautical maps provided by the Navy. You plan to bill the insurance investigator for 16 hours of work.

Demonstrate your use of the problem-solving process by completing the following.

A. Paraphrase the task and identify the information relevant to your task.

B. Represent this information in at least two of the three standard models: as a tables, graphs, or equations. Clearly show how the relevant given information relates to your model so that the insurance investigator can follow your reasoning.

C. Solve the problem and clearly document your work.

D. Interpret your solution by writing a brief conclusion for the insurance investigator. Use words like “north” or “west”.
Why: Statisticians, business analysts, and others will frequently need to model more than one set of data at the same time. These models can then be used to make comparisons and to make predictions.

Learning Objectives:
1. Multiple Representations:
   a. Model data with linear systems.
   b. Solve a system of equations using tables and technology.
2. Problem-solving: Make estimates using data.
3. Communication:
   a. Precisely and accurately define variables in a model and interpret solutions in a context.
   b. Explain the mathematical reasoning that supports an estimate or a conclusion.

Group Exercise:
1. The table below gives the semi-annual returns in dollars on two hypothetical investments starting on January 1, 2002.

<table>
<thead>
<tr>
<th>Years</th>
<th>0</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1500</td>
<td>1615</td>
<td>1730</td>
<td>1845</td>
<td>1960</td>
<td>2075</td>
<td>2190</td>
</tr>
<tr>
<td>B</td>
<td>2000</td>
<td>1920</td>
<td>1840</td>
<td>1760</td>
<td>1680</td>
<td>1600</td>
<td>1520</td>
</tr>
</tbody>
</table>

   a. Use the data given to estimate when the two investments had the same value. What are the investments worth at this point? Briefly explain your reasoning.

   b. For each type of investment, does value change linearly with respect to time? Support your answer.

   c. Find reasonable linear models for each investment. Briefly show your work or explain how you used your calculator. Clearly define your variables.

   d. Use your models to predict precisely when the investments had the same value. What is their value at this point? Use this answer to comment on the accuracy of your estimate in part a.
**Practice**

1. The table below gives the annual population in thousands for two cities.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>City A</td>
<td>13.6</td>
<td>22.0</td>
<td>33.1</td>
<td>43.7</td>
<td>50.2</td>
<td>52.9</td>
<td>57.4</td>
</tr>
<tr>
<td>City B</td>
<td>42.6</td>
<td>49.8</td>
<td>51.8</td>
<td>52.3</td>
<td>54.0</td>
<td>54.2</td>
<td>54.8</td>
</tr>
</tbody>
</table>

a. Use the data given to estimate when the populations of the two cities will be the same. Briefly explain your reasoning.

b. Plot the data. Use a different symbol for each city. Completely label your graphs.

c. Neither city has a population that is a perfect linear function of time, but the data for each city is roughly linear. Use the regression capabilities of your calculator to find a linear model for each city’s population. Clearly define your variables.

d. Based on your regression models, when were the populations of the two cities the same? Show your work or explain how you used your calculator. Write a sentence explaining the meaning of your solution in the context of this scenario.

e. Graph your regression lines and show the solution to part d on the graphs.
Intermediate Algebra

Unit 1 Activity 10: Unit Analysis

Why: Unit analysis is a powerful problem-solving method. For example, nurses use unit analysis to calculate dosages. In science classes, unit analysis is used to convert from one unit of measure to another. In advanced math classes, unit analysis is often used to check an answer --- if your answer has the right units, you most likely performed the correct calculations. You will see that unit analysis can also be used to guide your problem-solving strategy.

Learning Objectives:
Skills: Use unit analysis to convert from one unit to another or one rate to another.

Warm-up:
1. Unit analysis is based on multiplying fractions. To warm-up for unit analysis, simplify the following expressions. Give exact answers in reduced fraction form.

\[
\frac{2 \cdot 3}{3 \cdot 5} \quad \frac{6 \cdot 5}{25 \cdot 3} \quad \frac{8 \cdot 8 \cdot 3}{7 \cdot 9 \cdot 16} \quad \frac{2 \cdot 27}{9} \quad \frac{18 \cdot 2}{5}
\]

2. In unit analysis you treat the “units” as factors that can be simplified before multiplying. Simplify the following expressions. Give units with your answer. (Hint:)

\[
\frac{60 \text{ miles}}{\text{hour}} \cdot \frac{1 \text{ mile}}{2 \text{ hours}} \quad \frac{5280 \text{ feet}}{\text{mile}} \quad \frac{3 \text{ miles}}{1 \text{ hour}} \cdot \frac{5280 \text{ feet}}{\text{mile}} \cdot \frac{1 \text{ hour}}{60 \text{ minutes}}
\]

3. In unit analysis you will often use a “conversion factor”. A conversion factor is a fraction where the numerator and denominator are equal quantities. Fill in the missing pieces of the conversion factors.

\[
\frac{\_ \text{ hours}}{1 \text{ day}} \quad \frac{1 \text{ pint}}{\_ \text{ cups}} \quad \frac{\_ \text{ inches}}{1 \text{ foot}} \quad \frac{1 \text{ pound}}{\_ \text{ ozs}} \quad \frac{\_ \text{ meters}}{1 \text{ kilometer}}
\]

4. Which of the following fractions are equal to one?

\[
\frac{5}{12} \quad \frac{1 \text{ foot}}{12 \text{ inches}} \quad \frac{12 \text{ months}}{12 \text{ years}} \quad \frac{4 \text{ cups}}{1 \text{ quart}} \quad \frac{5280 \text{ feet}}{1 \text{ mile}} \quad \frac{1000 \text{ milliliters}}{1000 \text{ liters}}
\]

5. If you multiply a fraction by one, you don’t change its value. Unit analysis uses this idea to convert from one unit to another unit. If you multiply by a “conversion factor” you are really multiplying by one. Use the conversion factors to fill in the blanks of the unit analysis.

a. How many cups of liquid are in a 5-gallon container? 1 gallon = 4 quarts; 1 quart = 4 cups

\[
\frac{5 \text{ gallons}}{1} \cdot \_ \cdot \_ =
\]
b. How many miles is a 6-kilometer run?

\[
\frac{1}{1 	ext{ foot}} \cdot \frac{0.30 	ext{ meters}}{1 	ext{ foot}} \cdot \frac{1000 	ext{ meters}}{1 	ext{ kilometer}} \cdot \frac{5280 	ext{ feet}}{1 	ext{ mile}} = \]

**Group Exercise:** Use unit analysis to convert units. Include units in your answer. Round answers to the tenths.

1. The speed of light in a vacuum is 186,000 miles per second. Convert this speed to metric units.

   5280 feet = 1 mile, 1 meter ≈ 39.37 in.

2. If a paper clip weighs 3 grams, how many paper clips are in a pound?

   1 kilogram = 1000 grams, 1 kilogram = 2.2 pounds

   \[ \text{(Hint: Since you want "paper clips \ per \ pound" in the answer, set up the first fraction with \ "paper clip" in the numerator \ \frac{1 \text{ paperclip}}{3 \text{ grams}}) } \]

3. In the Northern Ireland, the minimum wage depends on the worker’s age. The minimum wage for youth aged 16 or 17 is £3 an hour; the minimum wage for workers aged 18 or over is £4.10 an hour. The minimum wage in California is $6.75 an hour. As of 9/28/04, one U.S. dollar was worth 0.5518 British pound.

   5) Where is the minimum wage higher, in California or in Northern Ireland?

   6) How much does a full-time, minimum-wage worker in California earn per year?

4. Last month I wrote down the following facts while on my delivery job: I worked 20 days during the month, my truck gets 21 miles per gallon, I drive at an average speed of 45 miles per hour, for 8 hours a day I listen to 104.5 on the radio, I drive about 35 miles per day, I work 5 days a week, gas costs $1.799 per gallon. How much did I spend on gas last month?
Practice:
1. A car going 70 mph goes how many feet in one second?
   *(Hint: what important conversion factor is missing?)*

2. A 15-oz bag of Yum-Yum Tortilla Chips contains 5 servings. There are 240 milligrams of salt in each serving. How many milligrams of sodium are in an ounce of these chips?
   *(Hint: If you want milligrams per ounce in the answer, set up the first fraction with milligrams in the numerator.)*

3. My shower is leaking at a rate of one drop per second. Should I be worried?
   a. How many cups of water per hour are being wasted?
   b. How many gallons per week?
   
   Useful conversion facts: 
   
   1 gallon = 4 quarts 
   4 cups = 1 quart 
   1 cup = 16 Tbsp 
   1 Tbsp = 3 tsp 
   1 tsp = 16 drops

   *(Hint: You want cups per hour, which is volume per time. So begin with the given information as a conversion fraction with volume (drop) over time (second).)*

4. During each surgery at a hospital 3 liters of plasma are used. Here are some more facts, many of which may be useful: 1000 milliliters are in a liter, one bag of plasma contains 1.5 liters, 15 drops are in each milliliter, one plasma bag costs $45, each surgery lasts five hours.
   a. How many bags of plasma are used per hour?
   b. What's the drip rate of the plasma (measured in drops per minute)?
   c. If 6 surgeries are performed per day, how much money is spent on plasma?